AP CALCULUS AB

SUMMER PACKET 2024 - 2025

CONWELL-EGAN CATHOLIC HIGH SCHOOL

This summer assignment is a review and exploration of key skills that are necessary for success in your 2024 - 2025 mathematics course as well as future high school mathematics courses.

All summer packets are due on the Friday of the first full week of school.

SOLVING EQUATIONS

Solve each equation.

1. $3x^2 - 8x - 19 = (x-1)^2$
2. $(3x-1)^2 = (2x+3)^2$
3. $3x + \frac{14}{5x} = \frac{31}{5}$
4. $\left(3x - \frac{2}{3}\right)^2 - 2 = \frac{5}{2}$
5. $\frac{2(x+3)^2}{3} - \frac{4}{9} = \frac{1}{3}$
6. $3x^2 + 6x = 2x^2 + 3x - 4$
7. $\frac{3}{4}(x-1)^2 = -\frac{4}{3}x + \frac{4}{5}$
8. $0.04x^2 - 0.2x + 0.25 = 1.21$

   (Complete the Square)
9. $x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{16}{9}$

   (Complete the Square)
10. $\frac{3}{4}(x-1)^2 = -\frac{4}{3}x + \frac{4}{5}$

    (Use the Quadratic Formula)
11. $6x^7 + 6x^5 = 9x^6 + 9x^4$
12. $x^6 - 64 = 0$
13. ~~$125x^4 - 27 = 125x^3 - 27x$~~
14. $e^{2x} + 5 = 12$
15. $\frac{1}{2}(10^{x+6}) - 5 = 14$
16. $4^{3-x} + 2 = \frac{5}{2}$
17. $\log_2 x + \log_2(x+4) - \log_2(x-2) = 4$
18. $x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32 = 0$
19. $\ln(2x-1) + \ln x = 0$
20. $e^{x-2} = 10^{2-x}$
21. $(5x+14)^{\frac{2}{3}} + 10 = 6$
22. $\sqrt{x+3} = 1 + \sqrt{x+1}$
23. $10 - 3\sqrt[3]{2x+5} = -11$
24. $\log_2(7x-8) - \log_2(x+1) - \log_2(x-1) = 1$
25. $4^{2x+3} = 5^{x-4}$
26. $\frac{\frac{4}{x-2}+2}{\frac{5}{x}-\frac{3}{x-2}} = 12$
27. $\frac{x}{x^2+4x-60} - \frac{2x}{x^2+6x-40} = \frac{1}{x^2-10x+24}$
28. $\frac{4x-3}{2x+2} = \frac{6x-3}{3x+5}$
29. $csc^2 x - \csc x - 2 = 0$
30. $3 - 2\cot x - 1 = 0$
31. ~~$2sin^2 x - \cos x - 1 = 0$~~
32. $\sin\left(x + \frac{\pi}{4}\right) + 1 = \sin\left(x + \frac{\pi}{4}\right)$
33. $\sec 2x = 2$
34. $|x^2 - 3x| = -4x + 6$
35. $x^4 - 4x^3 - 15x^2 + 58x - 40 = 0$
36. $u = c + d\ln x\,; solve\ for\ x$

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

46. Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

a. $f(2)$ b. $g(-3)$ c. $f(t+1)$

d. $f[g(-2)]$ e. $g[f(m+2)]$ f. $[f(x)]^2 - 2g(x)$

47. Let $f(x) = \sin(2x)$. Find the exact value of each.

a. $f\left(\frac{\pi}{4}\right)$ b. $f\left(\frac{2\pi}{3}\right)$

48. Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each

a. $h[f(-2)]$ b. $f[g(x-1)]$ c. $g[h(x^3)]$

49. Given $f(x) = \frac{1}{3}x + \frac{7}{4}$,

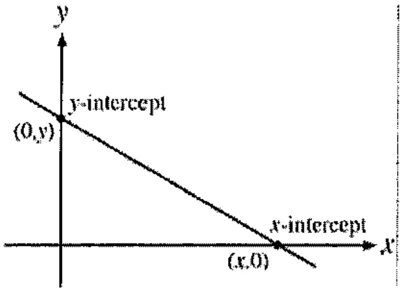
a. find the zeros of $f(x)$ b. solve $f(x) = \frac{1}{8}$ c. find $f\left(-\frac{9}{8}\right)$

50. Given $f(x) = -x^2 + x$,

a. find $f(0)$ b. find where $f(x) = 0$ c. find $f\left(-\frac{1}{3}\right)$

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

$x - \text{int.}\ (Let\ y = 0)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

x – intercepts $(-1,0)$ *and* $(3,0)$

$y - \text{int.}\ (Let\ x = 0)$

$y = 0^2 - 2(0) - 3$

$y = -3$

y – intercept $(0,-3)$

53. Find the x and y intercepts for each.

a. $y = 2x - 5$

b. $y = x^2 + x - 2$

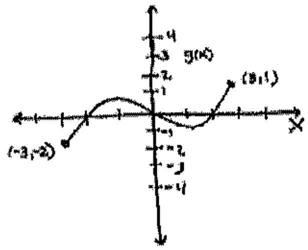
c. $y = x\sqrt{16 - x^2}$

d. $y^2 = x^2 + x - 2$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).

The domain is the set of inputs of the function.
Input values run along the horizontal axis.
The furthest left input value associated with a pt. on the graph is −3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is [−3, 3], that is all reals from −3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis.
The lowest output value of the function is −2. The highest is 1. So the range is [−2, 1], all reals from −2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

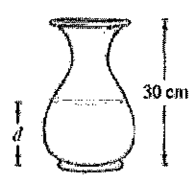
DOMAIN
For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2, 2]$

RANGE
The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0, \infty)$

55. Find the domain and range of each function. Write your answer in INTERVAL notation.

a. $f(x) = x^2 - 5$ b. $f(x) = 3\sin x$ c. $f(x) = \frac{-1}{|x-3|}$

56. Water runs into a vase of height 30 centimeters at a constant rate. The vase is full after 5 seconds. Use this information and the shape of the vase shown to answer the questions if d is the depth of the water in centimeters and t is the time in seconds. *(see figure)*



a. Explain why d is a function of t.
b. Determine the domain and range of the function.
c. Sketch a possible graph of the function.
d. Use the graph in part (c) to approximate $d(4)$. What does this represent?

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

Write each equation.

62. Determine the equation of a line passing through the point $(5, -3)$ with an undefined slope.

63. Determine the equation of a line passing through the point $(-4, 2)$ with a slope of 0.

64. Use point-slope form to find the equation of the line passing through the point $(0,\ 5)$ with a slope of $\frac{2}{3}$.

65. Use point-slope form to find a line passing through the point $(2, 8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

66. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point $(4, 7)$.

67. Find the equation of a line passing through the points $(-3, 6)$ and $(1, 2)$.

68. Find the equation of a line with an x-intercept $(2, 0)$ and a y-intercept $(0, 3)$.

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

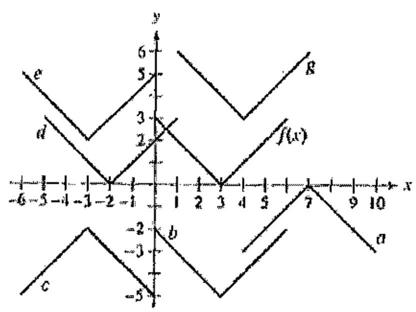
70. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How does the graph of $g(x)$ differ from $f(x)$?

71. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x –axis.

72. If the ordered pair $(2, 4)$ is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

a. $f(x) = 3$
b. $f(x-3)$
c. $f(x-2) + 1$
d. $2f(x)$
e. $-f(x)$
f. $f(-x)$

73. Use the graph of $y = f(x)$ to match the function with its graph.



a. $y = f(x+5)$
b. $y = f(x) - 5$
c. $y = -f(-x) - 2$

d. $y = -f(x-4)$
e. $y = f(x+6) + 2$
f. $y = f(x-1) + 3$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

75. **Determine all horizontal asymptotes.**

a. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

b. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$

c. $f(x) = \frac{4x^2}{3x^2-7}$

d. $f(x) = \frac{(2x-5)^2}{x^2-x}$

e. $f(x) = \frac{-3x+1}{\sqrt{x^2+x}}$

This is very important in the use of limits.

EVEN AND ODD FUNCTIONS

Recall:

Even functions *are functions that are symmetric over the y-axis.*

To determine algebraically we find out if $f(x) = f(-x)$

*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

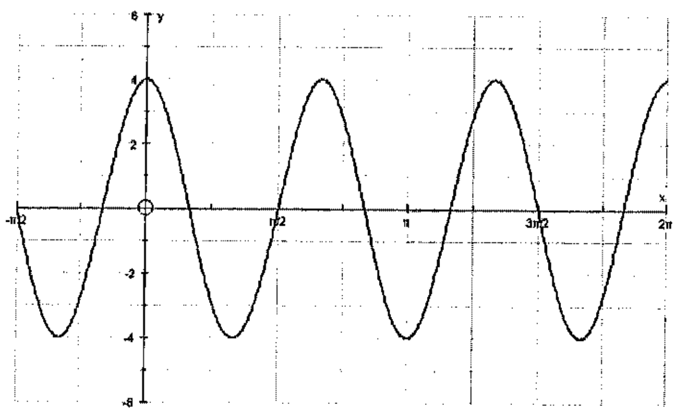
Odd functions *are functions that are symmetric about the origin.*

To determind algebraically we find out if $f(-x) = -f(x)$

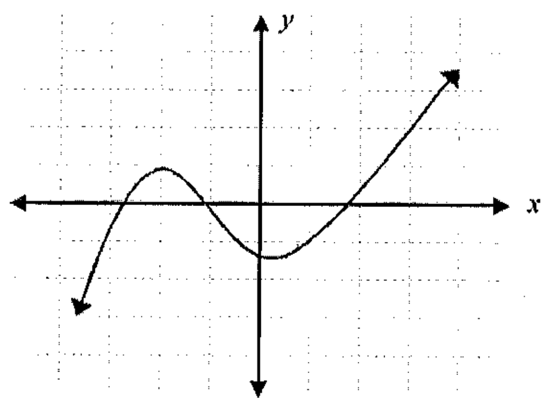
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

State whether the following graphs are even, odd, or neither. Show ALL work.

77.



78.



79. $f(x) = 2x^4 - 5x^2$

80. $g(x) = x^5 - 3x^3 + x$

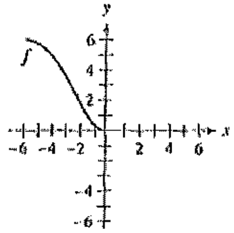
81. $f(x) = 2\cos x$

82. $k(x) = \sin x + 4$

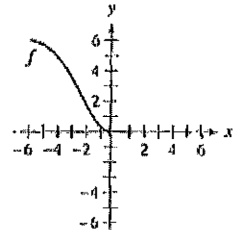
83. The domain of the function f shown in the figure is $-6 \le x \le 6$.

Complete the graph of f given that f is:

a. Even



b. odd



AVERAGE RATE OF CHANGE

86. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table below. Find the average speed of the car over each interval.

   a. 0 to 10 seconds
   b. 10 to 20 seconds
   c. 20 to 30 seconds
   d. 15 to 30 seconds

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	20	140	400	680	1400	1800

87. Water is draining from a large tank. After t minutes there are $160{,}000 - 8000t + t^2$ gallons of water in the tank.

   a. Find the average rate at which the water runs out in the interval from 10 to 10.1 minutes.
   b. Do the same for the interval from 10 to 10.01 minutes.
   c. Estimate the rate at which the water runs out after exactly 10 minutes.

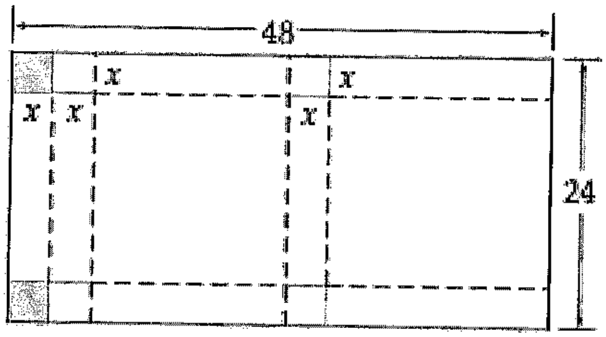
88. The volume of a cube can be found using the function, $f(x) = s^3$. Find the average rate of change of the volume of a cube whose side has length x and x changes from:

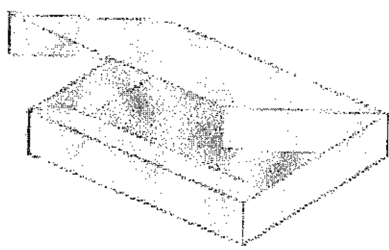
   a. 4 to 4.1 b. 4 to 4.01 c. 4 to 4.001.
   d. Estimate the rate of change of the volume at the instant when $x = 4$.

89. A rectangle has a perimeter of 100 inches, and one side has length x. Express the area of the rectangle as a function of x. Find the dimensions of the rectangle with perimeter 100 inches and the largest possible area.

90. A box with a square base has a volume of 867 in^3. Express the surface area of the box as a function of the length x of a side of the base. (Be sure to include the top of the box.). Find the dimensions of the box with volume 867 in^3 and the smallest possible surface area.

91. A box with a lid is to be made from a 48-by-24-inch piece of cardboard by cutting and folding, as shown in the figure. If the box must be at least 6 inches high, what size squares should be cut from the two corners in order for the box to have a volume of 1000 cubic inches?





92. A Quonset hut is a dwelling shaped like a cylinder. You have 600 square feet of material with which to build a Quonset hut.

a. The formula for surface area is $S = \pi r^2 + \pi r l$ where r is the radius of the semicircle and l is the length of the hut. Substitute 600 for S and solve for l.

b. The formula for the volume of the hut is $V = \frac{1}{2}\pi r^2 l$. Write an equation for the volume V of the Quonset hut as a polynomial function of r by substituting the expression for l from part (a) into the volume formula.

c. Use the function from part (b) to find the maximum volume of a Quonset hut with surface area of 600 square feet. What are the dimensions?